Executive Summary
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Problem B: Movement Of An Object In Microgravity Environments

We determine the range of dimensions for the smallest asteroid to land a probe on with a model that requires three parameters set by a space station: the velocity at which the probe would travel towards the surface of the asteroid, the density of the asteroid, and the volume of the asteroid. The model is obtained by setting the potential energy of the asteroid above the surface equal to its kinetic energy. We consider the mass of asteroid M, mass of the probe m, radius of the asteroid R, gravitational constant G, initial velocity of the probe v, volume of the asteroid V, r is the radius of the probe, d is the distance from the surface, and density of the asteroid ρ. Our resultant equation is:

$$\frac{GMm}{R} = \frac{1}{2}mv^2,$$

but $M = \rho V$, so the above equation, after simplification and if we assume it has spherical shape , becomes

$$\frac{8\pi}{3}G\rho R^3 - v_0{}^2 R - v_0{}^2(r+d) = 0$$

Using the fact that the velocity of the probe launched by Hayabusa traveled at 0.38 m/s², the density of Ryugu, the target asteroid, is 1270 kg/m³, it was launched from a distance d of 51 m from the surface of the asteroid, the radius of the probe is 0.15 m, and the fact that Ryugu is nearly spherical (and hence has spherical volume), by the model above, the radius of an appropriate model for the asteroid to land on, with the chosen target density at the chosen velocity, is 474.8 m. The actual radius of Ryugu is 450 m, which results in a percent error of 5.5%.

The best way to land the probe with the least amount of damage is to have the probe, or a contraption attached to the probe, deploy an airbag before its initial collision with the surface. We model the motion of the probe like that of a bouncing ball after its free-fall. After the initial impact, the probe will bounce, losing energy through kinetic frictional force after each successive bounce. We list the relevant terms:

k_f is the kinetic frictional constant, v_0 is the velocity before contact with the surface,

v_1 is the velocity of the rebound, L is the length of the unstretched spring, m is the mass of the probe,

k is the spring constant, T is the period of the spring, and $y(t)$ is the displacement of the probe from the surface where $[0,T]$ is the period of the spring.

We do not account for the spin of the probe while in motion or the normal force while it is in contact with the surface. The motion of the probe can be broken down into two distinct types of motion. The first is free-flight motion (not in contact with the surface), and the second is contact motion (in contact with the surface).

During free-flight, i.e., $y(t) > L$, the motion of the ball is described by

$$y''(t) = -g,\ x'(t) = v_{0x}$$

The equations above can be integrated and we find the solution,

$$y(t) = y_0 + v_{0y}\,t - \frac{gt^2}{2},\ x(t) = v_{0x}\,t,$$

where $y(0) = y_0 = L$, unless we are modeling the flight before the initial contact, $x(0)=0$,

and v_{0x}, v_{0y} are the vertical and horizontal components, respectively, of the initial velocity.

The motion of the ball when it is in contact with the surface, i.e., $y(t) \le L$, is described like an oscillating spring as follows:

$$(a)\quad my''(t) = k(L - y(t)) - mg,$$

$$(b)\quad mx''(t) = -k_f(k(L - y(t)) + mg),$$

The soutions to the above equations are

$$(a.1)\quad y(t) = v_{0y}\sqrt{\frac{m}{k}}\sin\left(t\sqrt{\frac{k}{m}}\right) + \frac{gm}{k}\cos\left(t\sqrt{\frac{k}{m}}\right) - \frac{gm}{k} + L$$

$$(b.1)\quad mx'(t) = -k_f(kL + mg)\,t + k_f k\int_0^t y(q)\,\mathrm{d}q + mv_{0x}t$$

$$(b.2)\quad mx(t) = -\frac{k_f}{2}(kL + mg)\,t^2 + k_f\cdot k\cdot\int_0^t\int_0^p y(q)\,\mathrm{d}q\mathrm{d}p + mv_{0x}t$$

With identical conditions as those in free-flight, we solve the integrals found in $mx(t)$ resulting in:

$$\int_0^t \int_0^p y(q)\,\mathrm{d}q\,\mathrm{d}p = -v_{0y}\left(\frac{m}{k}\right)^{\frac{3}{2}} \sin\left(t\sqrt{\frac{k}{m}}\right) - g\left(\frac{m}{k}\right)^2 \cos\left(t\sqrt{\frac{k}{m}}\right) + \frac{1}{2}\left(-\frac{gm}{k} + L\right)t^2 - \frac{v_{0y}gm}{k}t + g\left(\frac{m}{k}\right)^2$$

To use these equations (the free-flight and contact equations), we set time to be 0 and use the conditions from the previous contact equation and/or free-flight equation to set the next iteration for the equations, and iterate until the velocity is relatively small.

Finally, to move the probe to a predetermined position, an inward contracting spring launcher is used. Because of stabilizers on the bottom of the probe with larger radii and the fact that the probe is bottom heavy, the probe will land on the same side as it was launched from. A motor inside the probe compresses the internal spring some chosen distance c with a spring constant k as to account for the distance the probe must travel after the spring launcher is released and ejected out of the probe at an angle and velocity, chosen by the space station. This resulting velocity must be smaller than the escape velocity.

We set the equation for the potential energy of the compressed spring, equal to the kinetic energy. To choose the appropriate constants, k and c, the following equation must be satisfied:

$\sqrt{\frac{k}{m}}\,c = \left|\vec{v}_0\right| < \left|\overrightarrow{v_{escape}}\right|,\ where\ \left|\overrightarrow{v_{escape}}\right| \Rightarrow \frac{2GmM}{R}$, where c is the amount of compression.

If the above equation holds, then the motion of the projectile can be described by

$$x(t) = c\sqrt{\frac{k}{m}}\cos(\theta)\,t$$

$$y(t) = c\sqrt{\frac{k}{m}}\sin(\theta)\,t - \frac{1}{2}gt^2.$$

The minimum amount of energy required to move the probe to a predetermined position is preset by the space station choosing an appropriate spring constant k and compression c. These parameters will set a limit to the height and distance the probe can travel. Obstacles outside these limits will constrain the motion of the probe. However, this method of traversal allows for dependable expectations for minimum and maximum horizontal and vertical distance that could be traveled.

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